ISSN 1070-4272, Russian Journal of Applied Chemistry, 2015, Vol. 88, No. 5, pp. 871−878. © Pleiades Publishing, Ltd., 2015.

VARIOUS TECHNOLOGIES

Rheological Modeling of Polymeric Films under Loading[1]

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Received May 18, 2015

Abstract—The equations in this paper determine the kinetics of elastic deformation in polymeric films under loading to the point of destruction. The rheological model indicates that the destruction of polymeric films is characterized by relaxation time, initial stress and the level of limited deformation. These results make it possible to determine the critical time and allow prediction of the performance of this product. The destruction of polymeric films can be calculated from the results presented herein and only requires evaluation of the accumulated elastic deformation for each discrete load. The period of time after which the accumulated elastic deformation from discrete loading reaches a critical value provides a theoretical estimate of product life. This paper proposes a reasonable agreement between experimental data and model prediction.

DOI: 10.1134/S1070427215050237

Most polymer film is used to produce bags. These bags are used for packaging, storage and transportation of products. American and European patent applications relating to the production of plastic shopping bags date back to the early 1950s, but these were primarily composite constructions with handles fixed to the bag in a secondary manufacturing process. The modern lightweight shopping bag is the invention of Swedish engineer Sten Gustaf Thulin. The popularity of polymer bags can be traced to their convenience, light weight and capacity to be reused for other purposes.

Creep and destruction in polymer films has become a major concern during loading because of the viscoelastic nature of the polymer matrix, in which degradation is time-dependent upon the modulus. It has been shown that under uniaxial extension the rupture of polymer film can be categorized as dependent upon the applied load, molecular structure and the environment [1–3]. Typically, these bags are not durable. Most users have experienced sudden tearing of a loaded bag. This fairly frequent occurrence has prompted assessment of the efficiency of a bag under force loading for the duration of its use [4–10]. In-service bag rupture most often occurs near the handle. It is the most loaded section of the bag and experiences maximum tensile stress (Fig. 1). Overhead bag handles consist of two slit strips on each side of the bag that are welding together. The pressed film evenly redistributes the load acting on the width of the film (Fig. 1, section *b*). The most loaded section is A (Fig. 1) and this is the area where the handles rupture. These are usually overhead handles or other structures in areas immediately adjacent to them, where the polymeric film material experiences deformation in a horizontal direction. In reality, these deformed areas of material correspond to the kinematics of pure shear [11–15].

[1] The text was submitted by the authors in English.

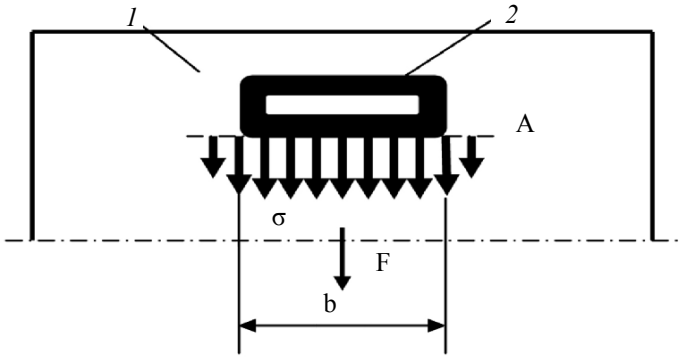

Fig. 1. A simplified diagram of the stress state of the loaded force *F*, (*1*) bag in the region adjacent to the handle, (*2*) arrows show the curve of tensile stresses σ in the most loaded section A.

Despite the increasing use of polymer films in load-bearing applications, published research on their durability is limited [9,10,12,13]. The present study was carried out because there is a lack of appropriate constitutive rheological modeling on the destruction of polymeric films under force loading.

RHEOLOGICAL MODELING

A nonlinear viscoelastic rheological model presented by Leonov [16] is used to describe the stress-strain behavior of polymeric films.

$$\begin{cases} \bar{\sigma} + p\bar{\delta} = 2\bar{c}W_1 - 2\bar{c}^{-1}W_2, \\ \bar{e}_f = \dfrac{1}{\theta_0(T)G_0(T)}\exp\left\{\alpha\psi\dfrac{I_1-3}{I_1-1} - \beta\dfrac{W^S}{G_0(T)}\right\}\times\left[\left(\bar{c} - \dfrac{I_1}{3}\bar{\delta}\right)W_1^S - \left(\bar{c}^{-1} - \dfrac{I_2}{3}\bar{\delta}\right)W_2^S\right] \\ \dfrac{d\bar{c}}{dt} + \bar{\omega}\bar{c} - \bar{c}\bar{\omega} - \bar{c}\left(\bar{e} - \bar{e}_f\right) - \left(\bar{e} - \bar{e}_f\right)\bar{c} = 0, \end{cases}$$

where $\bar{\sigma}$ is the stress tensor, p is the Lagrange multiplier, determined by the boundary condition, $\bar{\delta}$ is the identity tensor, $\bar{c}$ is the Cauchy strain tensor, $\bar{c}_f$ is the flow strain rate tensor, $\bar{\omega}$ is the vortex tensor, $\bar{e}$ is the strain rate tensor, β is the flexibility of macromolecular chains, ψ is dimensionless parameter (ψ = 0 at $\bar{\omega} = 0$ and ψ = 1 at $\bar{\omega} \neq 0$) characterizing the decrease of the activation energy of viscous flow of the polymer due to reversible destruction of its structure, α is dimensionless parameter (α = 1 at ω ≠ 0 and α = 0 at ω = 0) that characterize the presence or absence of reversible destruction of the structure of the polymer during deformation, $\theta_0(T)$ is the relaxation time, $G_0(T)$ is the tensile modulus, W is the strain energy function $W = W(I_1, I_2)$, I_1 and I_2 are the primary and the secondary strain tensor invariants, t is the time, $f(I_1, I_2)$ is the dimensionless function that defines relaxation time, and $2W^S = W(I_1, I_2) + W(I_2, I_1)$, is the symmetric function of W. The strain energy function parameters can be shown by:

$$W_1 = \frac{\partial W}{\partial I_1},\ W_2 = \frac{\partial W}{\partial I_2},\ W_1^S = \frac{\partial W^S}{\partial I_1},\ W_2^S = \frac{\partial W^S}{\partial I_2},$$

To describe the elastic properties of the polymer material in the rheological model (1), the following elastic potential which gives a fairly adequate results in a variety of kinematic types of polymer media under loading [13]:

$$W = 0.25G_0(I_1 + I_2 - 6). \quad (2)$$

In Eq. (2), the matrix of the kinematic tensor and tensor of elastic deformations within the rheological model (1) will be as follows:

$$\bar{e}_f = \dot{\varepsilon}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix};\quad \bar{\omega} = 0;\ \bar{c} = \begin{pmatrix} c & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & c^{-1} \end{pmatrix};\ \bar{c}^{-1} = \frac{1}{\det\bar{c}}\begin{pmatrix} c^{-1} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & c \end{pmatrix}$$

where $\dot{\varepsilon} \equiv \frac{1}{\lambda}\frac{d\lambda}{dt}$ is the rate of deformation of the polymer caused by the vertical force corresponding to the weight of the contents of the bag, $c \equiv \lambda_e^2$, λ_e, and λ are the elastic and total stretch ratio in the polymer, respectively. These parameters are defined based on Hencky strain in the following equations:

$$\lambda_e = \exp\left(\varepsilon_e^H\right),\ \lambda = \exp\left(\varepsilon^H\right), \tag{4}$$

where ε^H is the Hencky strain, ε_e^H is the elastic Hencky strain.

The primary and secondary invariants of tensor C are resulted from Eq. (3) as:

$$I_1 = I_2 = c + 1 + c^{-1}. \tag{5}$$

By utilizing Eqs. (2), (3), and (4), the following form of Eq. (1) can be developed.

$$\begin{cases} \bar{\sigma} + p\bar{\delta} = 0.5G_0(T)\begin{pmatrix} c-c^{-1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & c^{-1}-c \end{pmatrix} & (6) \\ \bar{e}_f \equiv \dfrac{d\bar{\varepsilon}_f^H}{dt} = \dfrac{1}{4\theta_0(T)}\exp\left[-\beta\left(c+c^{-1}-2\right)\right]\times\begin{pmatrix} c-c^{-1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & c^{-1}-c \end{pmatrix} & (7) \\ \dfrac{d\bar{c}}{dt} = 2\bar{c}\left(\bar{e}-\bar{e}_f\right) & (8) \end{cases}$$

where $\bar{\varepsilon}_f^H$ is the flow strain tensor as defined in Hencky.

It is possible to describe the development of stresses in a loaded polymeric film bag in the area under consideration and evaluate their performance for the duration of use under a given force loading. To solve this problem, some physical considerations are required. First, under these conditions, force loading of the material creates no normal stress along the plane of the film bag. It is possible using Eq. (6) to determine the expression for the Lagrange multiplier as:

$$p = 0.5G_0(T)(c^{-1} - c). \tag{9}$$

Second, the stretch ratio is defined as $\lambda = r(t)/r_0$ and the rate of deformation of a bag under constant external loading can be determined from Eq. (3) as follows:

$$\dot{\varepsilon}(\tilde{t}) \approx \frac{1}{\theta_0(T)}\frac{\tilde{\sigma}_0}{1+\tilde{\sigma}_0\tilde{t}}, \tag{10}$$

where $\tilde{\sigma}_0 = \dfrac{F}{S_0G_0(T)}$ is dimensionless initial stress, $\tilde{t} = \dfrac{t}{\theta_0(T)}$ is dimensionless time, F is the force acting on the bag (Fig. 1), S_0 is the cross- section of the bag in the treated area ($S_0 = 2b\delta$), δ is the thickness of a single layer of the film.

Substituting expressions (9) and (10) in tensor Eqs. (6) and (8), and taking into account Eq. (7) obtains the following scalar equations that describe the kinetics of elastic deformation and stress in a direction coaxial with the direction of loading:

$$\frac{dc}{d\tilde{t}} = 2c\left\{E(\tilde{t}) - \frac{\left(c-c^{-1}\right)}{4}\exp\left[-\beta\left(c+c^{-1}-2\right)\right]\right\}, \tag{11}$$

$$\sigma = G_0(T)\left(c-c^{-1}\right), \tag{12}$$

where $E(\tilde{t}) \equiv \dot{\varepsilon}(\tilde{t})\theta_0(T) = \dfrac{\tilde{\sigma}_0}{1+\tilde{\sigma}_0\tilde{t}}$ is dimensionless strain rate.

An analytical solution to differential Eq. (11) is not possible; therefore, an asymptotic solution can be determined using the following initial conditions:

$$\tilde{t} = 0\ :\ c = c_0. \tag{13}$$

Since the flexibility of the macromolecular chains of most polymers is not large ($\beta < 1$) and because large elastic deformation of a polymer does not accumulate ($c \to 1$), the following asymptotic analytical solution can be derived using Eq. (11):

when $\tilde{\sigma}_0\tilde{t} << 1$

$$c \approx \frac{\frac{\sqrt{1+4E_0^2}+c_0-2E_0}{\sqrt{1+4E_0^2}-c_0+2E_0}\left(\sqrt{1+4E_0^2}+2E_0\right)\exp\left(\sqrt{1+4E_0^2}\tilde{t}\right)+2E_0-\sqrt{1+4E_0^2}}{\frac{\sqrt{1+4E_0^2}+c_0-2E_0}{\sqrt{1+4E_0^2}-c_0+2E_0}\exp\left(\sqrt{1+4E_0^2}\tilde{t}\right)+1}, \quad (14)$$

when $\tilde{\sigma}_0 << 0{,}28$

$$c \approx \frac{(c_0+1)\exp(\tilde{t})+c_0-1}{(c_0+1)\exp(\tilde{t})-c_0+1}, \quad (15)$$

where $E_0 = E(\tilde{t}=0) \equiv \dot{\varepsilon}(\tilde{t}=0)\theta_0(T) = \tilde{\sigma}_0$.

RESULTS AND DISCUSSION

Analysis of the asymptotic analytical solution of Eqs. (14) and (15) shows that the function should have a maximum time to reach the value of E_0; however, if dimensionless time to the maximum value of elastic deformation is smaller than the characteristic relaxation time ($\tilde{t}<1$), then the maximum value of elastic deformation can be determined using Eq. (11) as follows:

$$c_{\max} \approx E_0 + \sqrt{E_0^2+1}. \quad (16)$$

It is clear from Eqs. (14), (15), and (16) that loading in this section of a plastic bag under permanent stress first increases and then reaches maximum value. The physical reason for this phenomenon involves two competing processes for the deformable polymer. Application of an external mechanical field (external loading on the plastic bag) causes elastic deformation of the polymer. This is because the free energy of the thermal motion of the kinetic units of its structure tend toward a specific rate (rate of relaxation) to restore the original structure of the deformed polymer. Under permanent loading, the strain rate decreases over time. After a certain period of time, the rate of the competing processes become equal and correspond to maximum attainable stress. The rate of relaxation prevails over the rate of deformation of the polymer. This causes elastic deformation to relax more rapidly than it accumulates, which decreases stress on the polymer. Destruction of the polymeric film occurs in the following situations:

– When force loading during use of the plastic bag causes the stress level of the tensile strength to reach the breaking point of the polymeric material; this depends on temperature and leads to destruction of the polymer film at a low strain level.

– Force loading during use of the polymeric article does not attain maximum tensile strength, but destruction of the item can result from a structural defect in the polymer in response to the development of large deformation.

In the second part, predicting the product life is possible only when the limit of deformation in the polymer is known [$\varepsilon_{\tilde{t}}^H(T)$]. As a first approximation, it can be assumed that the relationship is linear:

$$\left[\varepsilon_{\tilde{t}}^H(T)\right] = \left[\varepsilon_{\tilde{t}}^H(T)\right]_{\text{fix}} + k\left(\frac{\tilde{\sigma}_{0\text{fix}}}{\tilde{\sigma}_0}-1\right), \quad (17)$$

where $\left[\varepsilon_{\tilde{t}}^H(T)\right]_{\text{fix}}$ and $\tilde{\sigma}_{0\text{fix}}$ are the ultimate level of deformation and the corresponding dimensionless initial stress, respectively, and k is dimensionless coefficient.

In this case, the condition that determines the relative useful life of the product is:

$$\varepsilon_{\tilde{t}}^H(T) = \ln(1+E_0\tilde{t}_{\text{ex}}) \le [\varepsilon_{\tilde{t}}^H(T)] = [\varepsilon_{\tilde{t}}^H(T)]_{\text{fix}} + k\left(\frac{\tilde{\sigma}_{0fix}}{\tilde{\sigma}_0}-1\right). \quad (18)$$

During the considered scenario, the durability of the product is:

$$\tilde{t}_{\text{exp}} \le \frac{\exp\left(\left[\varepsilon_{\tilde{t}}^H(T)\right]\right)-1}{E_0} = \frac{\exp\left(\left[\varepsilon_{\tilde{t}}^H(T)\right]_{\text{fix}} + k\left(\frac{\tilde{\sigma}_{0\text{fix}}}{\tilde{\sigma}_0}-1\right)\right)-1}{\tilde{\sigma}_0}. \quad (19)$$

From the analysis of Eq. (19), it is known that destruction of product is characterized by relaxation time, initial stress and level of limited deformation. Fig. 2 shows the dependence of the dimensionless lifetime of film bags made of LDPE. This model is applicable to thermoplastic polymers which simultaneously possess a combination of two properties, viscosity and elasticity. The most of thermoplastic polymers are in this category. Therefore, as an example of practical use of theoretical results has been chosen the most typical representative of thermoplastics, LDPE. In the obtained model there is no any limit to use for any other viscoelastic thermoplastics.

The destruction of a bag is more of theoretical than practical interest. Users do not expect a long life for these bags. During the limited duration of usage, destruction does not usually occur, but the handles can be stretched several times. The first situation coincides more with actual experience, where the handles of the bag are torn during use because the stress exceeds the tensile strength $[\sigma(T)]$ of the polymer material. Here, destruction of the polymer material occurs in a relatively short time. Eq. (14) can calculate the durability of the product without significant error. The practical use of this dependency, which is common with Eq. (12), describes the kinetics of the development of stress in a polymeric film bag under loading. It is the only correct determination of c_0, which characterizes the level of elastic deformation in the material at the initial moment of exploitation of the polymer bag.

Such a formal approach to the problem is a significant distortion since, before the beginning of actual use of the product the loaded condition is always in the present moment, which is ignored. It is that short period of time to be determined by the process of loading the product, and only after the implementation of which actually begins the process of exploitation of the product. This process generally takes less than one second, but implementation of this process better defines the functionality of the product. This means that Eq. (14) is determined essentially by two seemingly independent parameters (c_0 and E_0). The relationship between these parameters will be as follows:

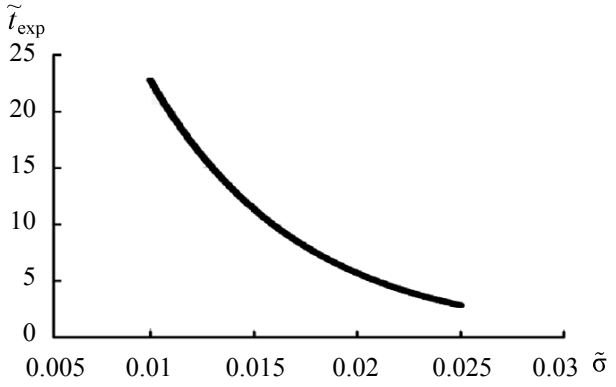

Fig. 2. The dependence of the relative durability of polymeric film bags versus the dimensionless initial stress. $G_0(T = 20°C) =$ 100 MPa; $\left[\varepsilon_i^H\left(T = 20°C\right)\right]_{fix} = 0.075$; $\tilde{\sigma}_{0fix} = 0.025$; $k = 0.1$.

$$c_0 = 1 + \frac{1}{2}E_0. \tag{20}$$

Substituting Eq. (20) into Eq. (14) yields Eq. (21):

$$c \approx \frac{\dfrac{\sqrt{1+4E_0^2}+1-1.5E_0}{\sqrt{1+4E_0^2}-1+1.5E_0}\left(\sqrt{1+4E_0^2}+2E_0\right)\exp\left(\tilde{t}\right)+2E_0-\sqrt{1+4E_0^2}}{\dfrac{\sqrt{1+4E_0^2}+1-1.5E_0}{\sqrt{1+4E_0^2}-1+1.5E_0}\exp\left(\tilde{t}\right)+1}. \tag{21}$$

Eq. (21) determines the kinetics of elastic deformations in the polymeric film bag during destruction. Substituting Eq. (20) into Eq. (11) defines the kinetics of stress development in the plastic film leading to destruction of the film in the handle area of the bag. These results make it possible to determine the critical time and predict performance of the bags. Defining critical stress (tensile strength at break) as the critical elastic deformation during destruction of the material can be determined from Eq. (12) as follows:

$$c_* = \frac{1}{2}\left(\frac{\left[\sigma(T)\right]}{G_0(T)} + \sqrt{\left(\frac{\left[\sigma(T)\right]}{G_0(T)}\right)^2 + 4}\right), \tag{22}$$

where c_* is the critical elastic deformation, and $[\sigma(T)]$ is the tensile strength at break.

Substituting Eq. (22) into Eq. (21) determines the maximum dimensionless time of product durability during which it cannot be destroyed:

$$\tilde{t}_{\exp} < \frac{1}{\sqrt{1+4E_0^2}} \times \ln\left\{ \frac{\sqrt{1+4E_0^2}+c_*-2E}{\sqrt{1+4E_0^2}-c_*+2E} \times \frac{\sqrt{1+4E_0^2}-1+1.5E}{\sqrt{1+4E_0^2}+1-1.5E} \right\}. \quad (23)$$

Equation (23) can be interpreted using previously obtained expressions [13]:

$$\tilde{\sigma}\left(\tilde{t}=0\right) \equiv \frac{\sigma\left(\tilde{t}=0\right)}{G_0\left(T\right)} \equiv \tilde{\sigma}_0 = \left(c_0 - c_0^{-1}\right) = E_0, \quad (24)$$

where $\tilde{\sigma}_0$ is dimensionless initial stress.

Equation (24) shows that during exploitation of the polymeric film bag, the initial dimensionless velocity of deformation in a product loaded with force F in the region of the handles equals the initial dimensionless stress occurring at the time of loading. The dimensionless time is shown in Eq. (23) to take the form:

$$\tilde{t}_{\exp} = \frac{t}{\theta_0(T)} < \frac{1}{\sqrt{1+4\tilde{\sigma}_0^2}} \times \ln\left\{ \frac{\sqrt{1+4\tilde{\sigma}_0^2}+c_*-2\tilde{\sigma}_0}{\sqrt{1+4\tilde{\sigma}_0^2}-c_*+2\tilde{\sigma}_0} \cdot \frac{\sqrt{1+4\tilde{\sigma}_0^2}-1+1.5\tilde{\sigma}_0}{\sqrt{1+4\tilde{\sigma}_0^2}+1-1.5\tilde{\sigma}_0} \right\}. \quad (25)$$

Note that the durability of polymer film bags under continuous loading using the first scenario of failure is determined force (F), cross-sectional area (S_0) of the polymer film, the physico-mechanical characteristics of the polymer that depend on temperature, and the rheological properties of the polymer (relaxation time and tensile modulus). The next step is to determine if Eq. (24) for continuous loading of a bag under real conditions for periodic loading is solved correctly. This problem can be solved theoretically using the results presented herein. All that is required is evaluation of the accumulated elastic deformation at each discrete loading. The period of time after which the accumulated elastic strain in discrete loading reaches a critical value is the theoretical estimate of the product life time. Figure 3 shows the dimensionless lifetime of film bags made of LDPE versus initial dimensionless stress.

It is clear that a LDPE bag is more effective when the dimensionless value of the initial stress is less than 0.05. The method of estimating the useful lifetime of the polymer film bag is simple. The critical and maximum elastic deformations attainable during use can be calculated from Eqs. (16) and (22), respectively. A comparison of these parameters obtains the durability of these bags. The $c_{\max} \geq c_*$ durability can be calculated from Eq. (25) and ($c_{\max} < c_*$) durability can be obtained from Eq. (19). The equations in this study allow the continuous equations for durability of polymer film bags without knowing the circumstances of their destruction. It is the destruction of the product that is important from a practical point of view. For the consumer, the reason why the bags tear is not important, but it is necessary to know how long the bag can carry a specific load. Fig. 4 shows a general portrayal of the relationship between results from Figs. 2 and 3. Such a generalized dependence enables manufacturers of polymer film bags with patch handles to produce different types of bags optimized for a particular range of operating loads. Manufacturers of such

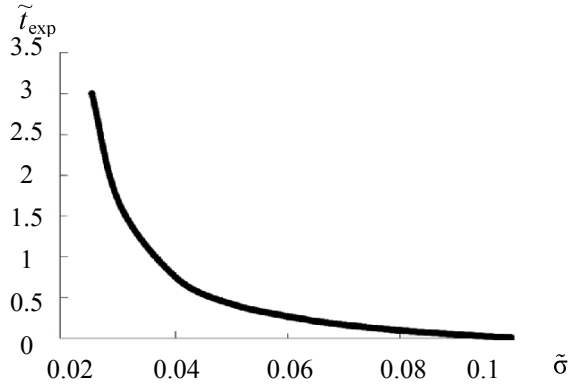

Fig. 3. The dependence of the relative durability of polymeric film bags versus the dimensionless initial stress. $G_0(20°C)$ = 100 MPa and $\sigma(20°C)$ = 10 MPa.

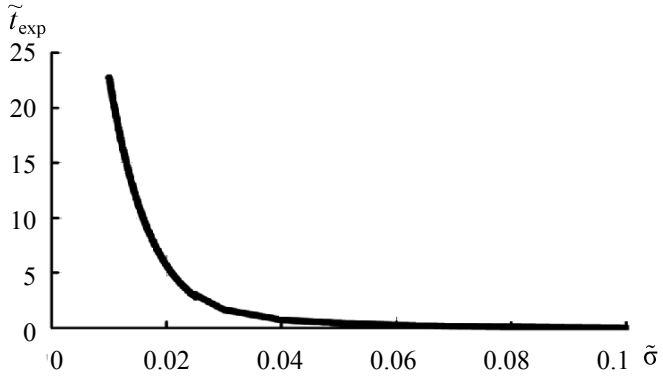

Fig. 4. Generalized dependence of the relative durability of polymeric film of LDPE bags on the dimensionless initial stress at T = 20°C.

bags could easily provide information about durability of the product (for size of load and longevity) that would allow consumers to determine its reliability and quality. It can be concluded from Fig. 4 that optimizing the composition and properties of the polymer material will increase the lifetime of the polymer bags by increasing the elastic shear modulus ($G_0(T) \to \max$) and decreasing creep (relaxation time ($\theta_0(T) \to \max$).

A problem arises when the relatively simple calculation of Eqs. (19) and (25) is the absence of information about the relaxation times of the polymeric materials. Without this characteristic, it is not possible to calculate the real lifetime; however, this problem has been simply solved with laboratory test methods for polymers. The desired relaxation time can be determined using a uniform uniaxial stretch test of the polymer and by evaluating the results of the reverse-effect. For this purpose, fast uniaxial stretching was done to achieve 3% to 4% elongation. Stretching was then stopped and deformation was maintained in the opposite direction to track the variation in length of time of load retention.

The experimentally-obtained equations can calculate the desired relaxation time of the polymer. For this purpose the homogeneous isothermal uniaxial stretching of a viscoelastic medium at a constant speed (V) was considered. The deformation of this medium corresponds to rheological model (1) and elastic properties were determined using Eq. (2). Taking into account these conditions, the kinematic tensors and tensor of elastic deformation in rheological model (1) will be:

$$\overline{e} = \dot{\varepsilon} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}; \ \overline{\omega} = 0; \ \overline{c} = \begin{pmatrix} c & 0 & 0 \\ 0 & c^{\frac{1}{2}} & 0 \\ 0 & 0 & c^{-\frac{1}{2}} \end{pmatrix}; \ \overline{c}^{-1} = \begin{pmatrix} c^{-1} & 0 & 0 \\ 0 & c^{\frac{1}{2}} & 0 \\ 0 & 0 & c^{\frac{1}{2}} \end{pmatrix}; \tag{26}$$

where $\dot{\varepsilon} \equiv \frac{1}{\lambda}\frac{d\lambda}{dt} = \frac{V}{l_0 + Vt}$, and l_0 is the initial length of the sample.

The invariants of tensor $\overline{c}$ are obtained from Eq. (26) as:

$$I_1 = c + 2c^{-\frac{1}{2}}, \ I_2 = 2c^{\frac{1}{2}} + c^{-1}. \tag{27}$$

Using expressions (26) and (27), the second equation of rheological model (1) will be:

$$e_f \equiv \frac{d\overline{\varepsilon}_f^H}{dt} = \frac{1}{4\theta_0(T)}\left[\left(\overline{c} - \overline{c}^{-1} + \frac{I_2 - I_1}{3}\overline{\delta}\right)\right] \times \exp\left[-\beta\left(c + c^{-1} - 2\right)\right]. \tag{28}$$

Neglecting the flexibility of the polymer chains and substituting Eqs. (26) and (28) into the last equation of the rheological model (1) obtains:

$$\frac{3\theta_0(T)}{c}\frac{dc}{dt} + c + c^{\frac{1}{2}} - c^{-1} - c^{-\frac{1}{2}} - 6\theta_0(T)\frac{V}{l_0 + Vt} = 0. \tag{29}$$

Eq. (29) describes the kinetics of elastic deformation of a polymer material under direct loading as obtained by stretching the sample. If stretching suddenly stops, the accumulated elastic deformation will relax and the force of Q will begin to decrease while maintaining the level of deformation. Eq. (29) is converted into the following form:

$$\frac{3\theta_0(T)}{c}\frac{dc}{dt} + c + c^{\frac{1}{2}} - c^{-1} - c^{-\frac{1}{2}} = 0 \tag{30}$$

An analytical solution of the differential Eq. (30) for small values of elastic deformation ($c \to 1$) is possible. For force (Q), the solution for relaxation time is:

$$\ln\left(\frac{Q}{Q_0}\right) = -\frac{t}{\theta_0(T)}, \tag{31}$$

where Q_0 is the force of maintaining the sample of deformation at cessation of stretching ($t = 0$).

Practical application of Eq. (30) that finds parameter $\theta_0(T)$, requires removal of the reverse-effect curve of the material to obtain Q over time. Practical use of Eq. (31) does not present difficulties. For example, if the reverse-effect curve obtained at room temperature of 20°C for an interval of 1 h is a decrease in stress from 103 to 100 N, expression (31) easily calculates θ_0 (T = 20°C) as 33.8 h.

CONCLUSION

It can be concluded that the durability of continuously-loaded polymer films can be determined by the force, cross-sectional area of the polymer film, physico-mechanical characteristics of the polymer and rheological parameters of polymers (relaxation time and tensile modulus). The results indicate that the formulation presented herein for a practical problem is specific and the solution can be obtained by understanding the rheology of polymeric materials. It can be concluded that it is possible to optimize the composition and properties of the polymer material to increase the lifetime of polymer films by increasing the elastic modulus ($G_0(T) \rightarrow \max$) and decreasing creep. The equations in this research enable manufacturers of polymeric bags to produce different types of bags optimized for a particular range of operating loads and also manufacturers of such bags could easily provide information about durability of the product that would allow consumers to determine its reliability and quality.

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